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Creative Informatics

Instructions

1. Do not open this brochure until the signal to begin is given.
2. Write your examinee ID below on this cover.
3. Answer three out of the four problems.
4. Three answer sheets are given. Use a separate sheet for each problem. You may use the backside of the sheet.
5. Write down the examinee ID and the problem ID inside the top blanks of each sheet.
6. Do not take out the sheets and this brochure from this room.

Examinee ID ______________________________

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Problem 1

Given a directed graph $G = (V, E)$, we would like to find *all-pairs shortest path lengths* which are the all shortest path lengths between every pair of vertices, where the size of the set V, $|V| = n$. Let e_{uv} denote a directed edge from a vertex u to a vertex v, and δ_{uv} denote the length of the edge e_{uv}. The graph G may have a negative length edge but does not have any negative length cycle. The length of the edge from the vertex u to the same vertex u, $\delta_{uu} = 0$, and when there exists no edge from the vertex u to the vertex v, $\delta_{uv} = \infty$.

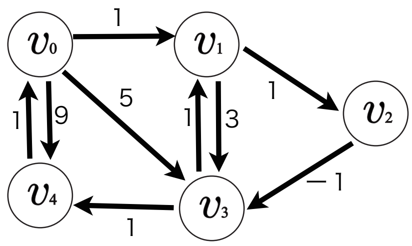

Figure 1: Graph $G_1 = (V_1, E_1)$

Algorithm 1 on the next page outputs the *single-source shortest path lengths*. Let $s \in V$ be a single source vertex, the shortest path length from the vertex s to a vertex $v \in V$ is stored in $d(v)$. Algorithm 2 outputs the all-pairs shortest path lengths table D, where the length of the shortest path from a vertex u to a vertex v is stored in $D(u, v)$. Each algorithm uses $d^{(k)}$ and $D^{(k)}$ $(k = 0, 1, \cdots)$ to store interim results, respectively. Answer the following questions.

(1) Apply Algorithm 1 to the graph $G_1 = (V_1, E_1)$ in Figure 1 to obtain the shortest path length from a single-source vertex v_0. Table 1 shows $d^{(0)}$ in Algorithm 1. Show the single-source path length $d^{(1)}$, $d^{(2)}$, $d^{(3)}$, and $d^{(4)}$ from the single-source vertex v_0.

(2) Apply Algorithm 2 to the graph $G_1 = (V_1, E_1)$ in Figure 1 to obtain the all-pairs shortest path lengths. Table 2 shows $D^{(0)}$ in Algorithm 2. Show the selected vertex $w \in V_1$ in the Main Loop and the corresponding table $D^{(1)}, D^{(2)}, D^{(3)}, D^{(4)}$, and $D^{(5)}$.

(3) To obtain all-pairs shortest path lengths, consider Algorithm 1-ALL which applies Algorithm 1 for all vertices in V as a single-source vertex. Compare Algorithm 1-ALL and Algorithm 2.

Table 1: $d^{(0)}$ in Algorithm 1

destination	
v_0	0
v_1	∞
v_2	∞
v_3	∞
v_4	∞

Table 2: $D^{(0)}$ in Algorithm 2

source\destination	v_0	v_1	v_2	v_3	v_4
v_0	0	1	∞	5	9
v_1	∞	0	1	3	∞
v_2	∞	∞	0	-1	∞
v_3	∞	1	∞	0	1
v_4	1	∞	∞	∞	0

Algorithm 1

```
for all $v \in V$ do
  $d^{(0)}(v) = \infty$
end for
$d^{(0)}(s) = 0$

/* Main Loop */
for $k = 1 \,..\, n-1$ do
  for all $e_{uv} \in E$ do
    $d^{(k)}(v) = \min(d^{(k-1)}(v),\ d^{(k-1)}(u) + \delta_{uv})$
  end for
end for
```

Algorithm 2

```
$k = 0$
for all $u \in V$ do
  for all $v \in V$ do
    $D^{(k)}(u, v) = \delta_{uv}$
  end for
end for

/* Main Loop */
for all $w \in V$ do
  for all $u \in V$ do
    for all $v \in V$ do
      $D^{(k+1)}(u, v) = \min(D^{(k)}(u, v), D^{(k)}(u, w) + D^{(k)}(w, v))$
    end for
  end for
  $k = k + 1$
end for
```

Problem 2

Answer the following questions.

(1) As Figure 1 shows, an orthogonal coordinate frame Σ_{C} of a camera with the lens axis CZ and the projection plane S is placed at the point C. The plane S is orthogonal to the lens axis CZ and has the distance f from C. The point Q is projected to the point P on the plane S with the coordinates $\mathbf{P}_{\mathrm{C}} = (P_X, P_Y, f)^t$ in Σ_{C}. The coordinates of three orientation vectors CX, CY, CZ are described as $\mathbf{X_W} = (X_X, X_Y, X_Z)^t$, $\mathbf{Y_W} = (Y_X, Y_Y, Y_Z)^t$ and $\mathbf{Z_W} = (Z_X, Z_Y, Z_Z)^t$, and the position vector of C is $\mathbf{C_W} = (C_X, C_Y, C_Z)^t$ in the coordinate frame Σ_{W}. The superscript ^{t} indicates transpose.

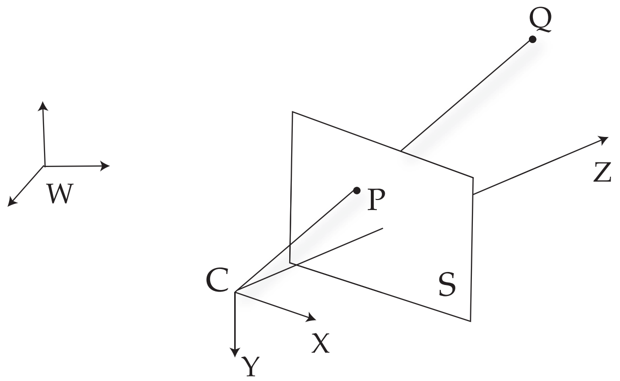

Figure 1

Assume the distance from C to Q is d, show the vector $\mathbf{Q}_{\mathrm{C}}$ from the point C to the point Q with $\mathbf{P}_{\mathrm{C}}$ and d. When the vector $\mathbf{Q_W}$ is the position vector of Q and the rotation matrix of Σ_{C} is R_{C} in Σ_{W}, we have $\mathbf{Q_W} = R_{\mathrm{C}}\mathbf{Q}_{\mathrm{C}} + \mathbf{C_W}$. Show the elements of the rotation matrix R_{C}.

(2) When we observe the point Q from the camera placed at a point A, the projection point is $\mathbf{P}_{\mathrm{A}} = (a_X, a_Y, f)^t$ in the camera coordinate frame Σ_{A}. Then, we translate the camera with the distance ℓ along the axis X to a point B and rotate it around the axis Y of the translated coordinate frame with the angle α. The rotated camera coordinate frame is Σ_{B}. The projection point becomes $\mathbf{P}_{\mathrm{B}} = (b_X, b_Y, f)^t$ in Σ_{B}.
Show the method to get the distance d_{A} from A to Q and the distance d_{B} from B to Q, when $\mathbf{P}_{\mathrm{A}} = \mathbf{P}_{\mathrm{B}}$ is obtained. Assume there is no error in the translation and rotation, and the XZ planes of Σ_{A} andΣ_{B} are aligned in the same plane.

(3) Two cameras are placed at the points M and N, respectively. Let the position vectors of M and N be $\mathbf{M_W}$ and $\mathbf{N_W}$ and the rotation matrices be R_{M} and R_{N}. The projection points of Q on these two cameras become $\mathbf{P}_{\mathrm{M}}$ and $\mathbf{P}_{\mathrm{N}}$. As the position vectors $\mathbf{Q}_{\mathrm{M}}$ and $\mathbf{Q}_{\mathrm{N}}$ of the point Q are the same in the coordinate frame Σ_{W}. Denote the condition which the projection points $\mathbf{P}_{\mathrm{M}}$ and $\mathbf{P}_{\mathrm{N}}$ should satisfy.

(4) Assume the projection points are described in an array and the condition in (3) is not satisfied. Let the evaluation function be $J = |(R_{\mathrm{M}}\mathbf{Q}_{\mathrm{M}} + \mathbf{M_W}) - (R_{\mathrm{N}}\mathbf{Q}_{\mathrm{N}} + \mathbf{N_W})|^2$, and consider minimizing J to get $\mathbf{Q}_{\mathrm{W}}$. Let d_{M} and d_{N} be the distances from M and N to Q, respectively, when J is minimized. Denote d_{M} and d_{N}. Then explain the method to get $\mathbf{Q}_{\mathrm{W}}$ in Σ_{W} with d_{M}, d_{N}.

(5) Explain the best arrangement to minimize errors when we measure a three dimensional position by two cameras such as (3).

Problem 3

Design a multiplier whose inputs are two 3-bit numbers and the output is a 6-bit number according to the following steps.

(1) Show the truth-table of the full adder and the half adder shown in Figure 1. Then construct them using AND, OR and NOT gates.

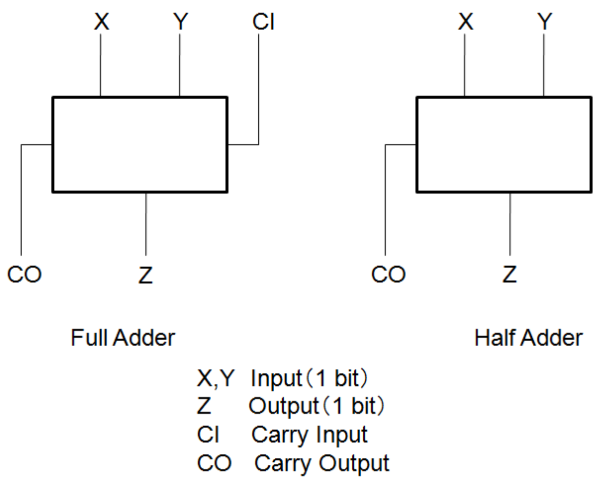

Figure 1: A full adder and a half adder

(2) Design the 4-bit adder shown in Figure 2 using the adders designed in question (1) with additional AND, OR and NOT gates.

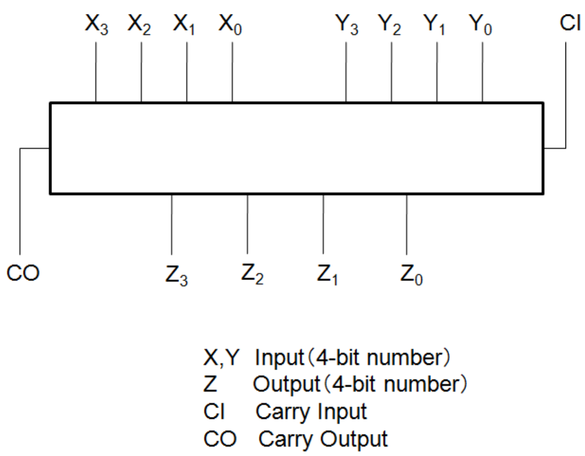

Figure 2: A 4-bit adder

(3) Design a 3-bit by 3-bit multiplier that produces 6-bit output using adders from (1) and (2) with additional AND, OR and NOT gates. Inputs for the multiplier are two unsigned 3-bit integers and the output is an unsigned 6-bit integer as shown in Figure 3.

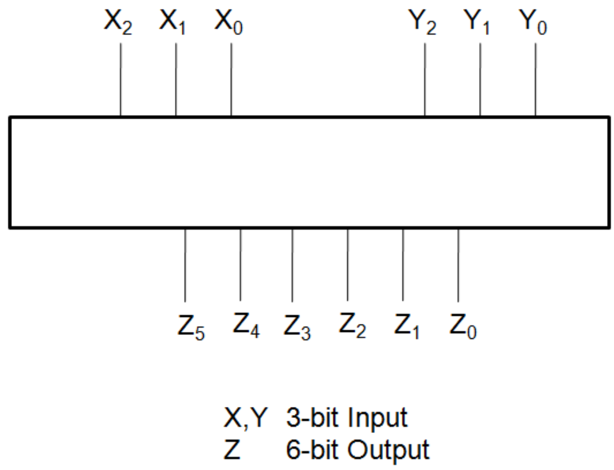

Figure 3: An unsigned 3-bit multiplier

(4) Design a 3-bit by 3-bit multiplier that produces 6-bit output using adders from (1) and (2) with additional AND, OR and NOT gates. The inputs of the multiplier are two signed 3-bit integers and the output is a signed 6-bit integer as shown in Figure 4. Two's complement numbers are used both in inputs and the output numbers.

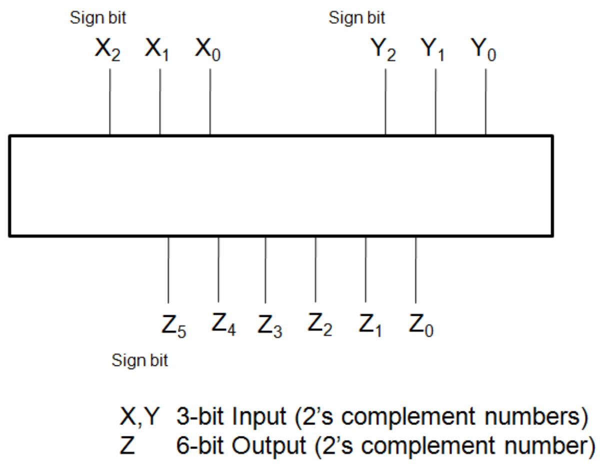

Figure 4: A 3-bit multiplier of signed integers

(5) Describe the construction of an N-bit by N-bit multiplier whose computation time is $O(\log N)$.

Problem 4

Select four items out of the following eight items concerning information systems, and explain each item in approximately 4∼8 lines of text, using examples or images if necessary.

(1) Bayes' theorem

(2) Decision tree learning method

(3) Spread-spectrum telecommunications and its applications

(4) Normalization in relational database

(5) Turing machine

(6) Snoop cache

(7) Unicode

(8) Three user authentication or personal identification techniques and a comparative analysis